AE 308

Modeling Errors in a Gothic Cathedral

Independent Study

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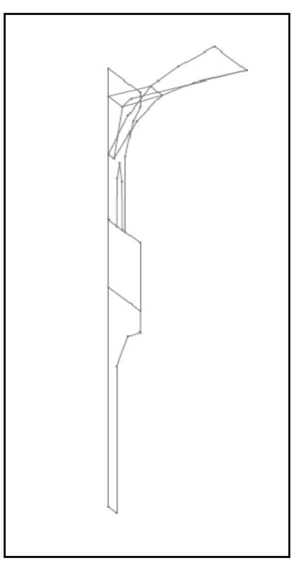
One of the most critical parts of the analysis of a structure is the modeling of it. Be it a computer model or a physical model, it is crucial that it accurately reflects its real-life counterpart. Done correctly, the model can be analyzed and the results applied to the actual structure.

An intriguing question involving the accurate modeling of structures is how the forces within a structure are affected by modeling errors. The focus of this study is on dimensional modeling errors and how exactly these errors alter the forces on and within a structure. The structure of interest is the vaults of the English Gothic Cathedral Beverley Minster, considered by many to be one of the most impressive churches in the country of England. Suppose an engineer visits this structure and inaccurately measures it. How exactly will this affect the model? In order to analyze this situation, a model of one bay of the main nave of the church will be constructed using correct survey data in the frame analyses program STAAD, and a second created with its dimensions altered by 10% in the transverse direction. The variations in the axial and shear forces carried by the vertical elements of the church will be investigated.

Using correct geometric dimensions taken from the field, one-fourth of the bay was first modeled. Necessary nodes were created, allowing the different members and materials to be added, according to the color-coded figure below. The three sets of ribs making up the vault were entered as beams, with their cross-sectional properties entered according to the data in Table 1. The surcharge was created as the only solid element of the model. The remaining elements were created as three and four-noded plates.



Two differing wall elements were created due to part of the wall containing intricate molding, and thus having different material properties. The material properties used for all elements of the structure are listed below in Table 1.

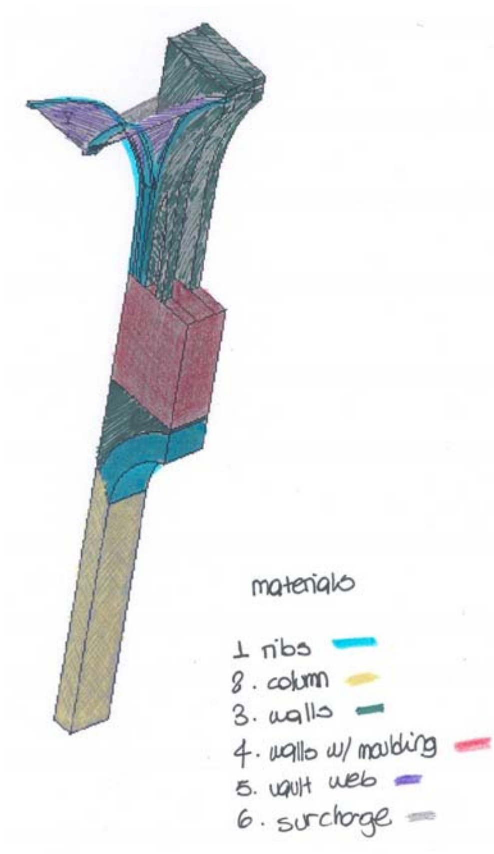
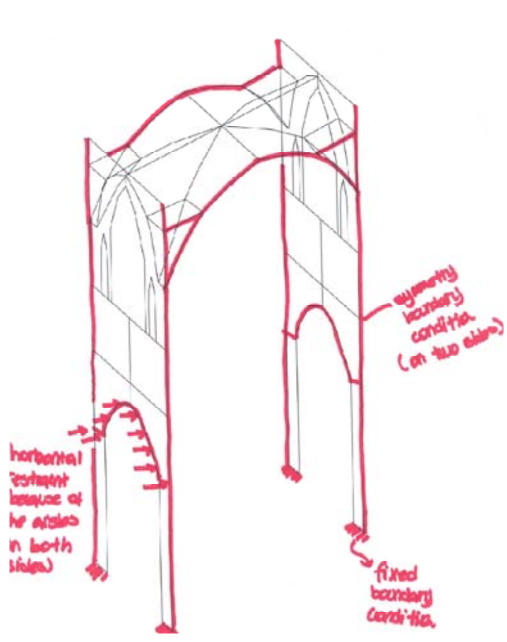


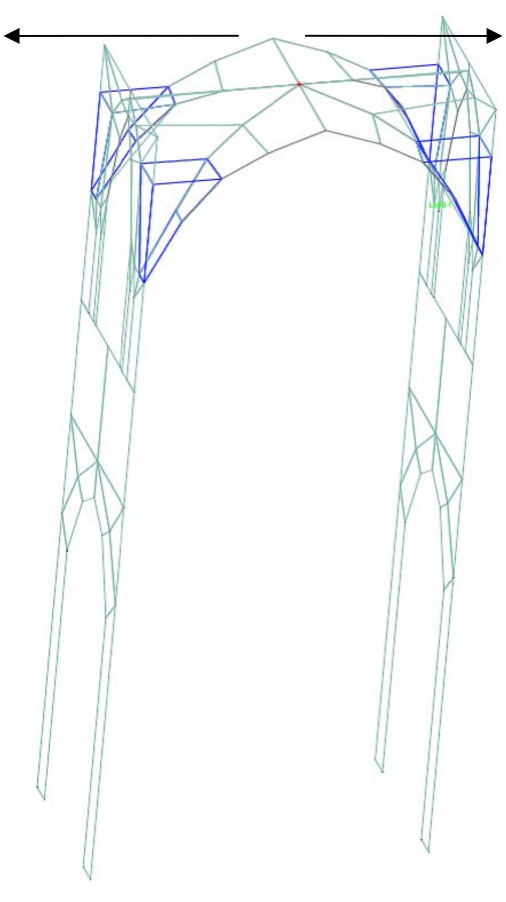
	Table 1		
	E	Density	Poisson's Ratio
Wall	10 GPa	2100 kN/m^3	0.15
Wall (molding)	5 GPa	2100 kN/m^3	0.15
Ribs	12 GPa	2100 kN/m^3	0.15
Column	15 GPa	2100 kN/m^3	0.15
Vault Web	8 GPa	2100 kN/m^3	0.15

With the material properties all entered, the model was then mirrored about two planes to obtain the entire bay. Because the bay is “cut out” of the rest of the church, boundary conditions are necessary. Horizontal reactions were added to portions of the bay to represent the forces present from the surrounding aisles. Additional boundary conditions restricting forces along the nave as well as the corresponding moment are added to the remaining two sides. Finally, the columns are “fixed” to the ground. With the addition of a gravity load, the structure is now accurately modeled and ready to be analyzed under its self weight.



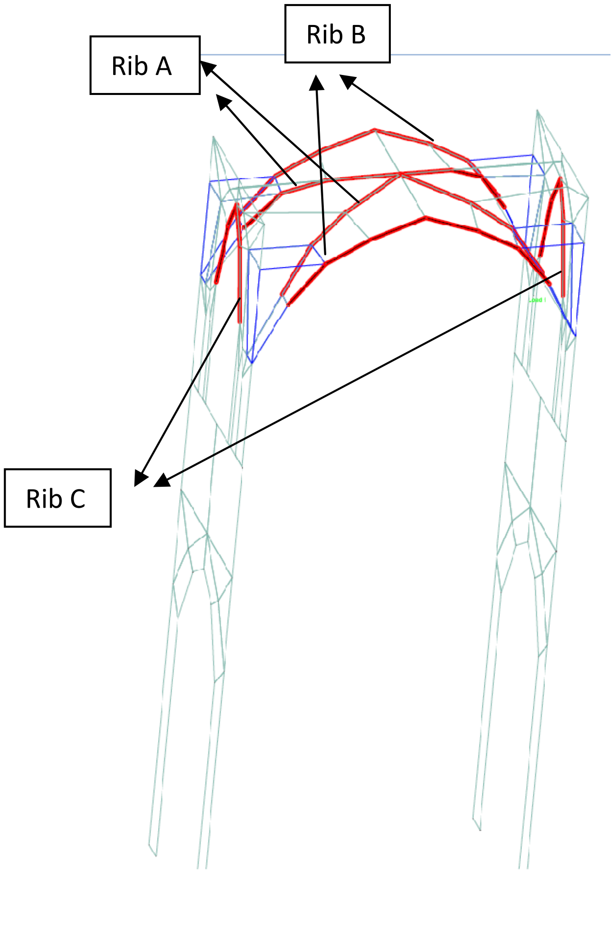
The objective of this study is to investigate the influence of modeling mistakes on the structural behavior of the vaults of the English Gothic Cathedral Beverly Ministry. With both a geometrically accurate computer model constructed and an additional model designed to be geometrically inaccurate constructed, the member forces in each can be analyzed and compared. The analyses will focus on the most critical components (ribs, arches, columns) of the nave, and will follow the general load path of the structure by analyzing the ribs and then the forces that eventually are carried to the structure's columns. Important displacements of the structure will also be compared.

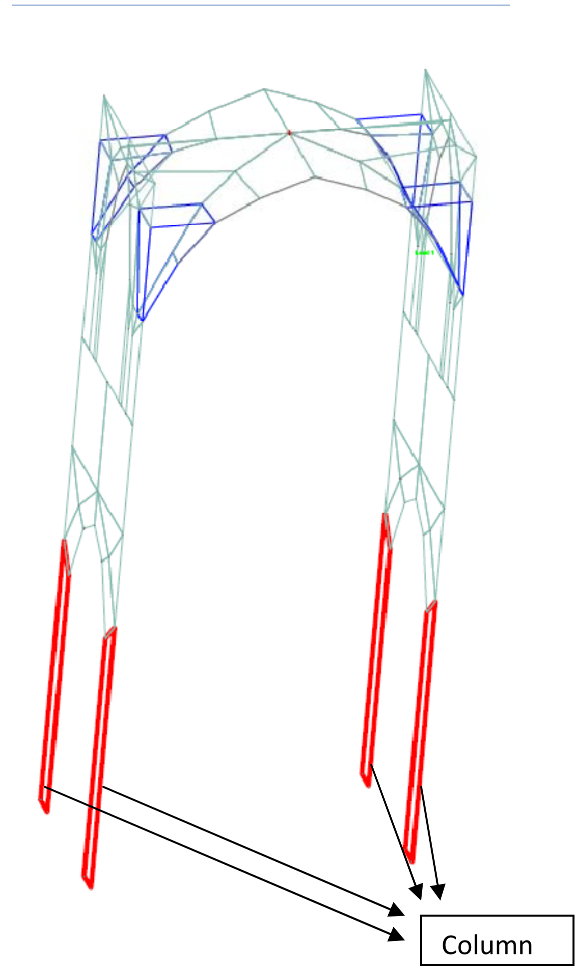
In order to create a model that contains modeling mistakes, a second model was constructed with its dimensions "stretched" 10% in the transverse direction, as is depicted in the image below. This model's results will be compared with that of the "geometrically accurate" version.



Structure "stretched" in direction of arrows

The first elements to be analyzed are the ribs. There are three different pairs of ribs that are analyzed. The maximum shear and axial forces in the ribs are compared in each model and the results are compiled in the table below. Almost every case resulted in the rib of the modified structure having more load than that of the correct model.





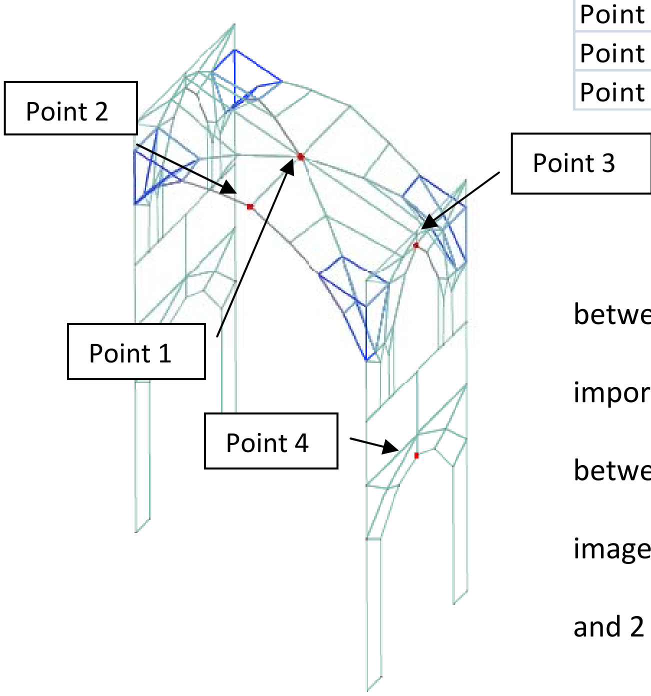
	Accurate Model		Inaccurate Model			
	Max. Axial	Max. Shear	Max. Axial	Max. Shear	% Increase in Axial Load	% Increase in Shear Load
Rib A	425 kN	78.9 kN	431 kN	85.4 kN	1%	8%
Rib B	377 kN	31.1 kN	418 kN	34.0 kN	11%	9%
Rib C	526 kN	35.3 kN	522 kN	36.5 kN	-1%	3%
Column	1880 kN	36510 kN	1888 kN	36833 kN	0%	1%

Moving on to the columns, one finds that the columns of the modified model have no real difference in loads from that of the correct model.

Perhaps the more interesting figure is the percent load increase in the incorrect model. The percent increase in axial and shear loads in the incorrect version ranges from -1% to +11%.

The average of these numbers is an increase of 4%.

	Accurate Model	Inaccurate Model	
	Deflection	Deflection (down)	Increase in Deflection
Point 1	14.0 cm	15.3 cm	9%
Point 2	14.6 cm	16.1 cm	10%
Point 3	9.87 cm	10.0 cm	1%
Point 4	6.20 cm	6.26 cm	1%

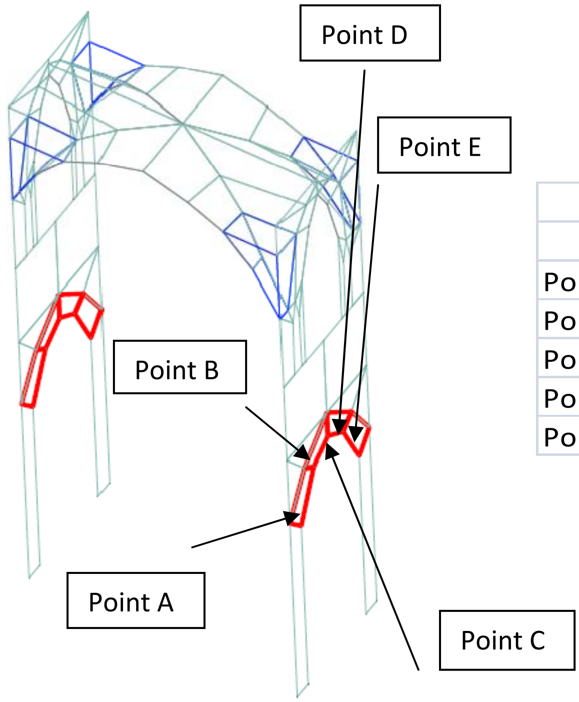


Deflection Points

Another interesting comparison to make between the two models is that of deflections. Four important nodes were selected to be compared between the two models and are displayed in the image above. As listed in the above table, Points 1 and 2 have an increase in deflection of approximately 10%, corresponding nicely with the 10% modification in dimensions. However, the remaining nodes have only an increase of 1%.

The last comparison between the two models that will be made concerns the boundary conditions corresponding to the aisles that run along both sides of the nave. These aisles create a force on the nave in the x-direction (into the nave), which was represented by applying boundary conditions along the arch. As shown in the figure and chart below, these support reactions all increase by an average of 18.2%.

So the question still remains, “How does a 10% error in dimensions affect the forces within the structure?” The two sets of ribs embedded in the vault webbing have an average



Aisles act on highlighted area of bay

	Accurate Model	Inaccurate Model	
	Force	Force	% Increase
Point A	1578 kN	1821 kN	15%
Point B	412 kN	503 kN	22%
Point C	5778 kN	6830 kN	18%
Point D	412 kN	486 kN	18%
Point E	1578 kN	1866 kN	18%

increase in axial and shear forces of 7.25%. The lower set of ribs experiences an average increase in forces of 2%, significantly less than the above ribs. Continuing down the structure, the columns experience even less of an increase, an average of 0.5%.

Similar results are found when comparing deflections between the accurate and erroneous model. The deflection in Points 1 and 2 (both part of the vault webbing) increases by 9% and 10% respectively in the modified model. Points 3 and 4 are located farther down the structure and have increases in deflection of only 1% each.

Therefore, it appears that when this model's geometric dimensions are increased by 10% in the transverse direction, forces within the members located near the top of the structure are also increased by approximately 10%. As elements increasingly closer to the base are analyzed, less and less of these increases in forces are seen, ultimately ending with the columns experiencing only a 1% increase in forces.

The reason for this may be found in the analyses of the reaction forces provided by the adjacent aisles, which were modeled as boundary conditions in this model. These reaction forces increased by an average of 18.2%, which is significantly more than the increases in the other analyzed members. Thus, it would appear that any of the additional forces that would be experienced by members located lower in the structure is “absorbed” by the surrounding aisles, leaving these lower members, such as the columns, without much of an increase in forces or deflections.

These results provide insight into how modeling mistakes affect the structural behavior of the Cathedral Beverly Minster. In this case, we assumed an engineer’s field measurements to be incorrect by a factor of 1.1 in a particular direction. Parts of the bay model experienced increases in load near 10%, while others experienced little to no change. Perhaps the most important result of this particular structure was the increase in forces on the adjacent aisles. A 10% dimensional modification results in almost a 20% increase in force on the adjacent aisles, a force that is not truly there. This modeling mistake is just as likely to happen in the opposite direction, resulting in a model that underestimates the forces on and from the aisles. These errors could prove to be critical, particularly in an aged building such as Beverly Minster, whose structural members and foundations may be crumbling away.